THE RESPONSE OF THE WEST ANTARCTIC ICE SHEET TO A REDUCTION OF RESISTIVE DRAG NEAR THE GROUNDING LINE

By

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Forward

The work for this thesis led into the writing of a paper which has been submitted for publication in the Journal of Geophysical Research. The main part of this thesis is the manuscript of the paper. Following the manuscript are appendices which show the derivations of the equations used in the model and the computer code for the calculations carried out in this work.

I thank my advisor, Ian M. Whillans, for his time and patience in working with me on this project. Many of our sessions over ice cream were of great importance as I worked toward the completion of this thesis. A special thanks goes to John Bolzan who was always willing to take time for valuable discussions regarding this work. Thanks also to J. Kostecka and R. Alley for their comments and those who served on my committee, J. Daniels and D. Merchant.

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Abstract

The hypothesis that the West Antarctic ice sheet is inherently unstable because its bed is below sea level (the marine ice sheet hypothesis) is tested using a one-dimensional ice-flow model. Based on continuity and a simplification of the flow law, the model follows a flow line from the ice divide, through an ice stream and into an ice shelf. Longitudinal stretching is taken to be constant with depth, and a vertically-averaged softness parameter is used. The boundary conditions are the current ice sheet, assumed to be steady, and specified longitudinal stress near the divide or calving front. Lateral flow convergence is specified and vertically-averaged velocities, longitudinal stretching stresses, driving stresses, and resistive drag (including basal drag) are calculated. Time-dependent inland ice profiles associated with a rise in sea level, are calculated by reducing the resistive drag near the grounding line. Different schemes for the redistribution of drag and resulting glacial responses are presented. Results show that local adjustments due to a reduction in resistive drag near the grounding line take on the order of 10^2 years.

Introduction

During the past thirteen years considerable attention has been given to the stability of the West Antarctic ice sheet. Conflicting interpretations have been advanced to explain the unusual surface elevation profile of this ice sheet. Hughes (1975), Thomas and Bentley (1978) and Mercer (1978) propose that West Antarctica is currently unstable and that a rise in sea level will cause the ice sheet to disintegrate resulting in a further rise in sea level of five meters (Mercer, 1978). Budd and McInnes (1979) and McInnes et al. (1983) state that West Antarctica appears to be recovering from a surge, and so is growing, rather than exhibiting characteristics preceeding a surge or disintegration. Still other workers, Whillans (1976) and van der Veen (1985), suggest that West Antarctica is stable and that no major changes have recently occurred or will take place.

It is the unique characteristics of the West Antarctic ice sheet that have created this growing interest in the ice sheet's stability. In contrast to East Antarctica and Greenland, the base of much of the West Antarctic ice sheet lies below sea level (Mercer, 1978). The drainage of West Antarctica is also mainly through ice streams, which are characterized by high velocities and plug flow and bordered by nearly stagnant ice, rather than by more nearly uniform flow throughout the ice sheet. It is the first of these characteristics that forms the focus of the investigation here: that a rise in sea level will float part of the grounded ice sheet off its bed.

The status of the West Antarctic ice sheet has been discussed in qualitative terms, or through models which include many interacting factors such as those by Thomas and Bentley (1978), van der Veen (1985), and Lingle (1984). These models all include a sliding relation. The model used here does not include a sliding relation but like the others is based on stress equilibrium-mass balance feedback, and includes the flow convergence into the ice streams.

The major effect of a rise in sea level or of climatic warming is held to be the partial flotation of formerly ground ice and a weakening of resistive drag at the grounding line or at some other site such as an island or shoal that impedes the flow. Reducing the resistive drag at a site causes a thinning of the ice sheet and an inland migration of the grounding line. This would then reduce the area of the bed available for resistive drag and cause an additional thinning. The process could be self-perpetuating, leading to a disintegration of the West Antarctic ice sheet.

The model used here tests the response of the ice sheet to such a reduction in resistive drag. Earlier models include various sliding relations which are considered somewhat speculative. It may be questioned that the calculated responses of the ice sheet are more associated with the sliding relation used than with the geometry of West Antarctica. In this model a sliding relation is not used, but instead, the resistive drag is specified, and the effects of a redistribution of resistive forces are calculated.

Methods

The adjustment of the ice sheet to a change, such as a rise in sea level, is studied by first constructing a model of the current ice sheet and then altering the distribution of forces on it. The forces are altered by decreasing resistive drag in the grounding zone (corresponding to flotation off the bed due to a sea level rise), and redistributing the stress elsewhere in order to maintain overall balance of forces. The adjustment of the glacier in response to these changes is studied. The startup position for the ice sheet is the condition today. Altering the values of resistive drag generates new ice sheet profiles.

Determining the Present Distribution of Resistive Drag:

The resistive drag is calculated from the geometry of the ice sheet and longitudinal stress gradients. The profile of the ice sheet has been measured and longitudinal stresses are calculated from strain rates, or velocity gradients, along the selected flow line. These velocity gradients are obtained by assuming the ice sheet to be in equilibrium so the velocity is just adequate to evacuate the annual accumulation. The annual accumulation is held constant at 0.1m/a. A flowline passing through ice stream B is used (Figure 1,2).

The equation of flow continuity used to calculate velocities is written as:

$$- \frac{\partial H\bar{U}}{\partial x} - \frac{H\bar{U}}{R} + \dot{b} = \frac{\partial H}{\partial t} \qquad (1)$$

in which $\bar{U}$ represents the average horizontal velocity through the thickness of the ice sheet, R is the radius of curvature of elevation contours, $\dot{b}$ the net annual accumulation and t is time. The use of the second term of equation (1) to describe horizontal lateral convergence or divergence is described fully in Van Heeswijk (1983). Equation (1) is solved for $H\bar{U}$ and then $\bar{U}$ along the flowline.

The flow law is used to link the velocity gradients, $(\partial\bar{U}/\partial X)$, obtained with stretching stresses (Paterson, 1983):

$$\frac{\partial \bar{U}}{\partial x} = A\tau_e^2 \sigma_x' \qquad (2)$$

σ_x' represents an appropriately weighted mean-longitudinal deviatoric stress. A is the softness parameter and τ_e^2 is the effective shear stress and is equal to $(\tau_{xx}^2 + \tau_{xz}^2)$. The full solution of this relation requires the shear stress which varies with depth. The present model does not include a vertical dimension and a simplified version of equation (2) is used.

In order to make the problem one-dimensional, the importance of shear stress in the flow law is neglected. The flow law is thus written as:

$$\frac{\partial \bar{U}}{\partial x} = A\sigma_x'^3 \qquad (3)$$

The simplification is valid if $\tau_e^2 \approx \sigma_x'^2$. For ice stream B this is a valid simplification (Whillans, 1986), and the simplification is also appropriate for most of the ice shelf. The value of A varies according to ice temperature and the relative role of shearing on horizontal planes. In this work, however, a constant value for A is used along the entire flow line.

Using the simplified flow law (3) the stretching stresses are calculated from velocity gradients and used to determine the resistive drag. The resistive drag is calculated from stress-equilibrium. It is described by Paterson (1983) as:

$$\tau_r = \tau_d + 2\frac{\partial \sigma_x' H}{\partial x} \qquad (4)$$

$$\tau_d = -\rho g H \frac{\partial h}{\partial x} \qquad (5)$$

where τ_r is the resistive drag, ρ is the density of ice, g is acceleration due to gravity, H is the ice thickness, x is the horizontal distance from the divide and σ_x' is the longitudinal deviatoric stress. The driving stress,

τ_d, as the equation shows is readily calculated from the measured geometry of the ice sheet. The resistive drag is thus equal to the driving stress plus an adjustment for differential pushes and pulls along a flow line as described the last term in equation (4).

Figure 3 shows the calculated distribution of resistive drag along the flowline from the ice divide to the calving front. Where the ice is grounded most of the resistance occurs as basal drag. For the ice shelf the resistance occurs at islands and shoals or as side drag. The grounding line is determined by bouyancy of the ice in sea water. As in van der Veen's model (1985), this method allows for a smooth transition from the inland ice through the ice stream to the ice shelf across the grounding line. A major difference is that in this work shear on horizontal planes, which is important in the inland ice, is not considered here.

Response to a Change in Resistive Drag:

It is possible to vary and redistribute the resistive stress as calculated above for the present day ice sheet, and to study the ice sheet's response. The equations as previously described are used in the reverse order to determine future ice sheet profiles. Here the distributed resistive drag, τ_r, is specified and the ice sheet thickness changes with time are calculated.

From the distributions of resistive drag and driving stress new stretching stresses are obtained using the equation of stress-equilibrium (4). The stretching stress in turn is used to calculate the stretching rate of the ice. From the stretching rate velocities are determined, and by using these in the equation of continuity, the change in ice thickness with time can be calculated.

The thicknesses obtained after each time step are used in the equation of stress-equilibrium and the computational cycle is repeated. The resistive drag distribution remains unchanged after the start of the forward calculation, but because the ice thickness and driving stresses change, the stretching stresses are affected. In turn the velocity gradients, velocities and $\partial H/\partial t$ are affected. This set of calculations is thus stepped through time to describe the adjustment of the ice sheet. If stable, the ice sheet will find a new configuration of balance ($\partial H/\partial t = 0$).

Numerical Techniques:

Tables 1 and 2 are flow charts which describe the solution of the set of equations used in the model. The technique of finite differencing is used for the numerical solution.

To achieve better numerical stability the expression ($\partial H\bar{U}/\partial X$) from the equation of continuity is split into its components $H(\partial\bar{U}/\partial X) + \bar{U}(\partial H/\partial X)$. If the full expression ($\partial H\bar{U}/\partial X$) is used, calculations at even and odd grid points are independent of one another and results show waves along the flowline of wavelength twice the grid spacing.

In the forward calculation, or "start-up" procedure, calculations are made by averaging values along grid nodes horizontally. In the reverse calculations it is necessary to average both horizontally and over time. In order to calculate new thicknesses (H') after a time step, velocities ($\bar{U}'$) and stretching stresses (σ_x'') at the new time must be known. An iterative technique is used for each time step to make such a calculation, using present-day velocities, thicknesses, and stretching stresses as the initial conditions for this iterative procedure (Table 2).

A time step of one year and a distance step of 5km are sufficient for numerical stability. σ_x' is calculated half-way between grid nodes and all other values are on the grid nodes.

Discussion

A decision on the distribution of forces must be made in applying the model. This is necessary because the sum total of forces acting on the glacier must at all times equal zero. If the resistive drag at one site is altered, there must be a compensating change in a force elsewhere to maintain net force balance. This compensation could occur with a change in resistive drag in some other place, with a change in longitudinal force acting at the ice divide or with a change in the force from the ice shelf. The model as described above does not indicate which compensation is appropriate, but some possibilities are more attractive than others.

A less attractive version is an immediately increased tension at the ice divide in response to a reduction in resistive drag near the grounding line. This means that part of the driving force that was formerly balanced by resistive drag is held instead by extra longitudinal tension all the way to the ice divide. This tension causes immediate thinning of that entire region. We calculate this rapid transmission of stress to be unrealistic.

Even less attractive is a model with immediately increased compression or buttressing from the ice shelf to compensate for the reduced resistive drag near the grounding line. This would result in immediate growth so that it contacts more islands or other obstructions.

A local redistribution of forces seems to be more realistic. If the drag just up-glacier from the grounding line is reduced this requires a nearby increase in resistive drag. Various versions of this are possible and we have elected to increase the resistive drag immediately up-glacier from the region where it was reduced. Resistive drags down-glacier are very small and immediate increases there are perhaps less likely.

The effect of reducing drag near the grounding line and increasing it up-glacier is shown in Table 3. Initially the stretching stresses are increased in the region of reduced and increased resistive drag. This leads to increased stretching and thinning. The geometry of the glacier adjusts until the new driving stresses approximately balance the new distribution of resistive drag. As indicated in Table 3, this adjustment occurs in about 200 years.

This readjustment introduces a second order problem. As the geometry of the glacier changes, the driving stress and net driving force change and for force balance there must be some further force compensation. This causes secondary effects to occur elsewhere, either on the ice shelf, inland ice, or both. It is not important where the secondary effects occur, as shown by two sets of calculations which allowed for the two extreme possibilities of force compensation. The timescale of change and the final ice sheet profiles for these two experiments are virtually the same (Table 3) and so this second order effort can be neglected.

The response of the ice sheet is rapid (Table 3). Part of the explanation for this is that the changes are small and thus do not take long to affect the ice sheet. The adjustments also occur locally, which enables the ice sheet to respond faster than if adjustments were to traverse the length of the ice sheet.

Of significance is that although the reduction of resistive drag covers a zone of 115 km, the grounding line retreats by only 45 km. This indicates that there is a feedback effect that works against disintegration of the ice sheet. To test this further, a second reduction of resistive drag after 200a is applied to the ice sheet (Figure 3). This represents inland migration of the ungrounding zone. The ice sheet adjusts in a similar manner to the first reduction. The response is rapid and small.

The stabilizing feedback effect is the requirement of gross force balance to be effected by local adjustment in resistive drag. For this model, the ice is sufficiently soft that it adjusts its geometry such that the driving stress locally balances resistive drag, except for small unimportant departures due to longitudinal stress gradients. Thus the major changes in geometry occur quickly and mainly at the sites where resistive drag is altered. The bed slope, as noted by earlier authors, is not sufficiently reversed that grounding line retreat is unstable. We have tried a number of redistributions of basal drag and always find rapid adjustment to a near stable configuration that is not radically different from the present ice sheet.

The next level of discussion is the mechanics of determination of resistive drag. We have not attempted that here but prior workers have used sliding law to describe that. We suggest that possible instabilities in West Antarctica are not likely associated with these mechanics.

For the above experiments the softness parameter, A, and annual accumulation, $\dot{b}$, are held constant for the entire flowline ($A = 1.8 \times 10^{-25} s^{-1} Pa^{-3}$, $b = .1$ m/a). Calculations using a value of A for ice 20°C warmer

(0°C) show that a time step of .1 year rather than 1 year is needed to maintain numerical stability, and the response of the ice sheet is greater. For softer ice the longitudinal stretching stresses are smaller and τ_r more closely equals τ_d. As τ_r is changed the warmer ice can respond more quickly because it is softer and is not so strongly supported by longitudinal stresses. Doubling the value of $\acute{b}$ creates larger longitudinal stresses and a slightly longer response time. These studies do indicate, however, that the model is not highly sensitive to the values of A and $\acute{b}$.

Conclusions

By incorporating force balance and continuity into a model of a marine ice sheet it is found that changes in resistive drag near the grounding line have small effects on the ice sheet. Response is rapid and the ice sheet appears to adjust in a stable manner to a reduction in resistive drag near the grounding line. Although the response time scale is different, van der Veen (1985) obtained similar results in his model of a marine ice sheet.

The adjustments of the ice sheet occur locally. This is because the surface slopes change until the driving stresses approximately equal the resistive stresses. Although the longitudinal stretching stresses may be large the longitudinal force gradients become unimportant after the ice sheet has adjusted to the changes.

The effects observed are dependent on the area of changed resistive drag. If the redistribution of resistive drag covers a larger area, then the ice sheet adjustments will occur over this greater area. There is little or no effect up-glacier and down-glacier of the changes and any effects in these areas are secondary effects dependent on the change of the net driving force.

In these studies the increased drag is placed just up-glacier from the reduction in drag. If there is a gap between these changes in drag (if the increased drag is placed much farther inland) more ice is drained from the ice sheet. This is still not of enough significance to cause catastrophic thinning of the ice sheet.

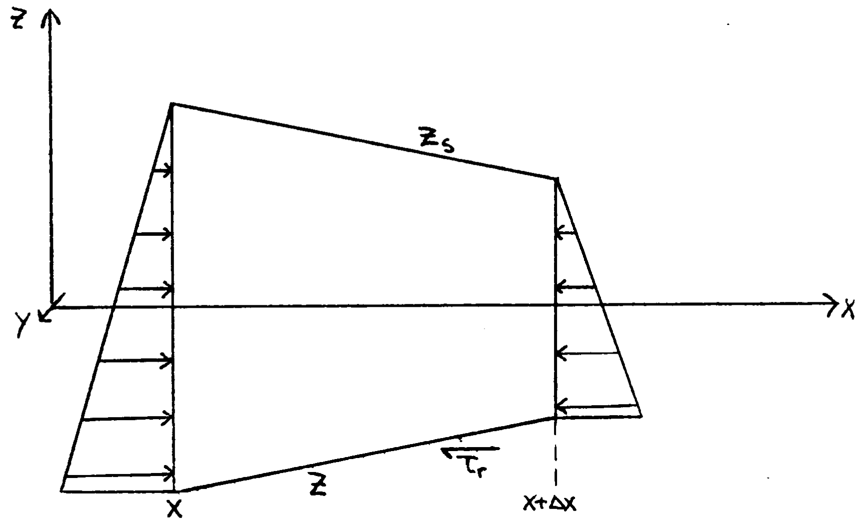
This model has limitations. Because shearing is neglected in the flow law, calculations are most nearly valid for areas of plug-flow such as the ice streams and ice shelf. The model is one-dimensional and changes in the flowline and lateral convergence are not considered.

The model presented here is one which does not include the mechanics that determine the distribution of resistive drag on an ice sheet, unlike those of Lingle (1984) and van der Veen (1985). Because these mechanics are not yet clearly understood this model emphasizes the importance of determining just where a compensating force for a reduced resistive force should be placed. Varying the locality and area of force redistribution leads to slightly different results but all results point to an ice sheet which responds stably to a reduction of resistive drag near the grounding line.

Appendix I

Derivations:

Stress Equilibrium:



Consider a block of ice with width, w, surface elevation, Z_s, bed elevation, Z_b, and resistive stress, τ_r. In an ice sheet acceleration can be considered negligible, therefore, from Newton's Law the sum of the forces should equal zero:

$$\sum F = ma = 0$$

In this model the forces acting on the ice sheet which are taken into consideration include the resistive stress at the bed times bed area, and tectonic force and lithostatic forces acting on the faces at x and $x+\Delta x$. the sum of these forces should equal zero.

$F = -\tau_r \cdot w\Delta x$ — resistive stress•area

$-\sigma^T(x)\cdot w\cdot(Z_s(x)-Z_b(x))$ — tectonic force on left

$+\int_{Z_b(x)}^{Z_s(x)} \rho g(Z_s(x)-Z_b(x))\cdot w\cdot dz$ — lithostatic force on left

$+\sigma^T(x+\Delta x)\cdot w\cdot(Z_b(x+\Delta x)-Z_b(x+\Delta x))$ — tectonic force on left

$-\int_{Z_b(x+\Delta x)}^{Z_s(x+\Delta x)} g(Z_s(x+\Delta x)-Z_b(x+\Delta x))\cdot w\cdot dz$ — lithostatic force on right

$-\rho g(\bar{Z}_s-\bar{Z}_b)(Z_b(x+\Delta x)-Z_b(x))\cdot w$ — force of bed

$=0$

ρ is the density of ice and g the acceleration due to gravity. (Z_s-Z_b) is the mean thickness of the section and $(Z_b(x+\Delta x)-Z_b(x))$ is the portion of the bed facing the force.

Let $H=Z_s(x)-Z_b(x)$

$H+\Delta H=Z_s(x+\Delta x)-Z_b(x+\Delta x)$

$Z_b=\Delta Z_s-\Delta H$

The balance of forces then becomes:

$$-\sigma^T(x)\cdot H+\rho gH^2+\sigma^T(x+\Delta x)(H+\Delta H)-\tfrac{1}{2}\rho g(H+\Delta H)^2-\rho g((H+H+\Delta H)/2)\Delta Z_b$$
$$-\tau_r\Delta H=0$$

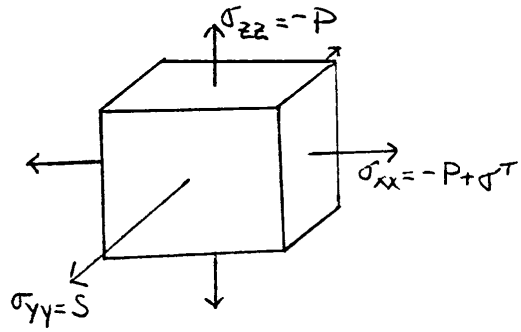
Dividing by Δx, subsitituting ΔZ_s-H for Z_b, and simplifying yields:

$$-\tau_r+((\sigma^T(x+\Delta x)(H+\Delta H)-\sigma^T(x)(H))/\Delta x)-g(H+\Delta H/2)Z_s=0$$

By taking the limit of this as $\Delta x\rightarrow 0$ the equation can be expressed as:

$$-\tau_r + \frac{\partial(\sigma^T H)}{\partial x} - \rho g H \frac{\partial h}{\partial x} = 0$$

The deviatoric stretching stress σ_x' needs to be limited to the full stretching stress. In this model the stretching stress is needed rather than the tectonic stress. The full stresses acting on a small block are as shown. σ_{ij} is the full stresses, σ^T, the tectonic stress, -P, the lithostatic pressure, σ_x' is the deviatoric stress, and S is the spherical stress.



$$\sigma_{ij}' = \sigma_{ij} - S$$

and

$$S = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

The stress components acting on the block are

$$\sigma_{yy} = S$$
$$\sigma_{zz} = -P + \sigma^T$$
$$\sigma_{zz} = -P$$

by substitution

$$S = \frac{1}{3}(-P + \sigma^T - P + S)$$

simplification yields

$$S = -P + \frac{1}{2}\sigma^T$$

The deviatoric stress may be written then as

$$\sigma_x' = \sigma_{xx} - S = -P + \sigma^T - (-P + \frac{1}{2}\sigma^T)$$
$$= \frac{1}{2}\sigma^T$$

Because the tectonic stress is twice the deviatoric stress, σ_x', and representing Z_s by h, the equation of stress equilibrium may be written as follows:

$$-\tau_r = 2\frac{\partial(\sigma_x' H)}{\partial x} - \rho g H \frac{\partial h}{\partial x}$$

The resistive drag, τ_r, and driving stress, $-\rho g H\frac{\partial h}{\partial x}$, approximately balance each other. The difference of the two stresses is balanced by a gradient of the stretching stress multiplied by thickness over a horizontal distance.

Flow Law:

The flow law is an empirically derived equation which relates the stresses acting on the ice sheet with the strain rate (Paterson, 1983).

$$\dot{\epsilon}_{ij} = A \tau_e^{n-1} \sigma'_{ij}$$

$$\tau_e^2 = \sigma'^2_{xx} + \tau^2_{xz}$$

A is a temperature dependent softness parameter, $\dot{\epsilon}$ is the strain rate, σ'_x is the deviatoric stress,and τ_e is the effective shear stress. It is determined that n=3. This model is one-dimensional and does not include a vertical dimension, so τ_{xz} is neglected and the effective shear stress is expressed as

$$\tau_e = \sigma'_{xx}$$

The flow law for this case is written as

$$\dot{\epsilon}_{xx} = A \sigma'^2_{xx} \sigma'_{xx}$$

The strain rate $\dot{\epsilon}_{xx}$ is the longitudinal velocity gradient:

$$\dot{\epsilon}_{xx} = \frac{\partial \bar{U}}{\partial x}$$

$\bar{U}$ is vertically averaged velocity and x is the horizontal distance. The flow law for this model is written as:

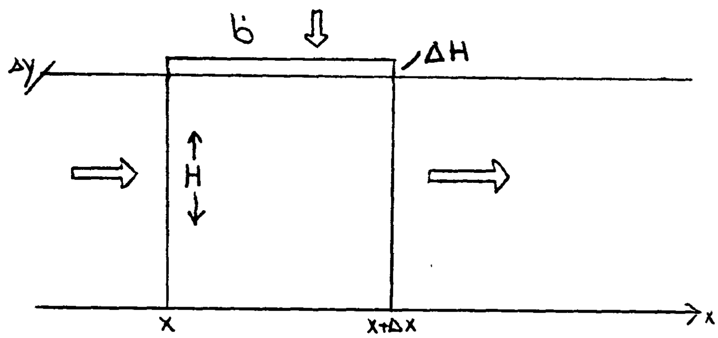
$$\frac{\partial \bar{U}}{\partial x} = A \sigma'^3_x$$

Increasing the value of A can approximately compensate for the neglected shear stress. The flow in this form is a one-dimensional expression which relates horizontal stretching stresses to horizontal strain rates.

Continuity:

The equation of continuity is derived from the principle of conservation of mass. Over a cross-sectional area the inputs are added and the outputs subtracted to yield the change in thickness over time.

For this derivation it is assumed that in map view the flow lines are parallel. This neglects any effects due to funneling and reduces the problem to one-dimension. In the final equation a lateral convergence term has been added as described by M. Van Heezwijck (1983).



Let b be the annual accumulation, $\bar{U}$, the average velocity, H, the thickness of ice, and Δy be the width. The mass flux into the box minus the mass flux out should equal change in thickness with time. The fluxes into and out of the box are summed and set equal to thickness change with time.

$$\rho \Delta y\, H(x)\bar{U}(x) - \rho \Delta y\, H(x+\Delta x)\bar{U}(x+\Delta x) + \rho \Delta y\, \dot{b}\Delta x = \frac{\rho \Delta y \Delta x H}{\Delta t}$$

The density, ρ, and width, Δy, divide out because they are constant. Dividing by Δx yields:

$$\frac{H(x)\bar{U}(x) - H(x+\Delta x)\bar{U}(x+\Delta x)}{\Delta x} + \dot{b} = \frac{\Delta H}{\Delta t}$$

By taking the limit as $\Delta x \to 0$ the equation my be expressed as $-\frac{\partial H\bar{U}}{\partial x} + \dot{b} = \frac{\partial H}{\partial t}$

Adding lateral convergence or divergence to the equation yields:

$$-\frac{\delta H\bar{u}}{\delta X} - \frac{H\bar{u}}{R} + \dot{b} = 0$$

R is the radius of curvature of surface contours.

If the difference of the flux into a section from upflow and the flow out of the section equal the annual accumulation the the section will be in equilibrium and $\frac{\partial H}{\partial t}=0$. If they are not equal then there will be a change in thickness with time which is explained by the equation of continuity.

Startup Computer Code:

```
//TS1304 JOB 'SDX350,328706683','DONNELLAN',
// REGION=512K,TIME=(0,20)
/*JOBPARM LINES=10000
//STEP1 EXEC FORTVCG,PARM.FORT='LANGLVL(77)'
//FORT.SYSIN DD *
C
C  STARTUP AD851015
C
C  THIS PROGRAM COMPUTES VALUES OF BASAL DRAG USING GIVEN THICKNESS
C  AS THE INPUT DATA
C
      IMPLICIT  DOUBLE PRECISION(A-H,O-Z)
      DIMENSION H(1,900),B(900),U(1,900),XKM(900),RGH(1,900),
     +SIGX(1,900),TAUB(1,900),RGHB(1,900),SIGXB(1,900),X(900)
      DIMENSION R(900),RKM(900),BED(900),S(900),TAUBB(1,900),SF(900)
C
C  VARIABLES ARE DEFINED AS FOLLOWING:  H=THICKNESS,M; B=ACCUM
C   RATE,M/A; U=VELOCITY,M/A; VEL=VELOCITY,M/S; XKM=HORIZ
C   DIST,KM; X=HORIZ DIST,M; RGH=RHO*G*H,PA; RGHB=RGH,BARS;
C   SIGX=DEVIATORIC STRESS,PA; SIGXB=SIGX,BARS; TAUB=BASAL
C   DRAG,PA; TAUBB=TAUB,BARS; BED=BED ELEV,M; S=SURFACE ELEV,M
C
C  CONSTANTS ARE DEFINED AS FOLLOWING:  A=FLOW LAW CONST,SE-1*PAE-3;
C   SPA=SEC/YEAR,S; PA=100000BARS; RHO=ICE DENSITY,KG/ME3;
C   G=GRAVITATIONAL ACCEL,M/SE2; DELX=STEP DIST, 100KM
C
C  ASSIGN VALUES FOR CONSTANTS
      SPA=3.15D+07
      A=1.8D-25
      PA=100000.D+00
      RHO=917.D+00
      G=9.78D+00
      DELX=5000.D+00
      WRITE(6,112)DELX
  112 FORMAT('1',' DISTANCE STEP EQUALS ',F8.1,' METERS'//)
C
C  ASSIGN LOOP VARIABLE
      N=1025000.D+00/DELX+1.D+00
      M=N-1
C
C  LABEL HEADINGS FOR OUTPUT
      WRITE(6,100)
  100 FORMAT(' CALCULATION OF "STARTUP" VALUES'//)
      WRITE(6,101)
  101 FORMAT(' X(KM)       H(M)            R(KM)     AVERAGE VELOCITY',
     $'     DEVIATORIC STRESS    DRIVING STRESS     BASAL DRAG')
      WRITE(6,102)
  102 FORMAT(34X,'(M/A)',16X,'(BAR)',14X,'(BAR)',11X,'(BAR)'/)
C
C  FILL XKM AND B ARRAYS
C  CALCULATE THICKNESS AND RADIUS OF CURVATURE
      STEP=0.D+00
      DO 1 I=1,M
      B(I)=0.1D+00
      X(I)=DELX*STEP
```

```
      XKM(I)=X(I)/1000.D+00
C   CALCULATE SURFACE PROFILE
      SINT=2130.73963753D+00
      SX1=-0.00116124D+00
      SX2=-1.1693194D-07
      SX3=7.4316170D-13
      SX4=-2.0245300D-18
      SX5=2.6264798D-24
      SX6=-1.3227578D-30
      S(I)=SINT+SX1*X(I)+SX2*(X(I)**2.D+00)+SX3*(X(I)**3.D+00)+
     $SX4*(X(I)**4.D+00)+SX5*(X(I)**5.D+00)+SX6*(X(I)**6.D+00)
      IF(X(I).GT.525000.D+00) S(I)=-.1058D-03*X(I)+140.1D+00
C   CALCULATE BED PROFILE
      IF (X(I).LT.50000.D+00) THEN
        BINT=2.2343674D-11
        BX1=-0.02120098D+00
        BX2=2.2354167D-06
        BX3=-9.8762255D-11
        BX4=1.0906863D-15
        BED(I)=BINT+BX1*X(I)+BX2*(X(I)**2.D+00)+BX3*(X(I)**3.D+00)+
     $  BX4*(X(I)**4.D+00)
      ENDIF
      IF(X(I).GE.50000.D+00) BED(I)=-1000.D+00
      IF(X(I).GT.125000.D+00) BED(I)=.02D+00*X(I)-3500.
      IF(X(I).GT.150000.D+00) THEN
        BINT=20276.27011274D+00
        BX1=-0.40475457D+00
        BX2=3.1371758D-06
        BX3=-1.2514167D-11
        BX4=2.7092624D-17
        BX5=-3.0199322D-23
        BX6=1.3563988D-29
        BED(I)=BINT+BX1*X(I)+BX2*(X(I)**2.D+00)+BX3*(X(I)**3.D+00)+
     $  BX4*(X(I)**4.D+00)+BX5*(X(I)**5.D+00)+BX6*(X(I)**6.D+00)
      ENDIF
      IF(X(I).GT.397000.D+00) BED(I)=-700
      SF(I)=BED(I)*(1.0D+00-1025.D+00/RHO)
      H(1,I)=S(I)-BED(I)
      IF(S(I).LE.SF(I)) H(1,I)=S(I)/(1.D+00-RHO/1025.D+00)
C   CALCULATE RADIUS OF CURVATURE
        RINT=-1024550.08443342D+00
        RX1=8.00708578D+00
        RX2=-0.00014123D+00
        RX3=2.8648497D-09
        RX4=-2.3618092D-14
        RX5=8.5037776D-20
        RX6=-1.1514543D-25
        R(I)=RINT+RX1*X(I)+RX2*(X(I)**2.D+00)+RX3*(X(I)**3.D+00)+
     $  RX4*(X(I)**4.D+00)+RX5*(X(I)**5.D+00)+RX6*(X(I)**6)
      IF(X(I).GE.275000.D+00) R(I)=-1000000.D+00
      IF(X(I).GE.300000.D+00) R(I)=-2000000.D+00
      IF(X(I).GE.325000.D+00) R(I)=-20000000.D+00
      IF(X(I).GE.350000.D+00) R(I)=-200000000.D+00
      R(I)=R(I)/2.D+00
      RKM(I)=R(I)/1000.D+00
      STEP=STEP+1.D+00
 1    CONTINUE
      BED(N)=2.D+00*BED(M)-BED(M-1)
      S(N)=2.D+00*S(M)-S(M-1)
      R(N)=-200000000.D+00/2.D+00
```

```
      RKM(N)=R(N)/1000.D+00
      X(N)=DELX*STEP
      XKM(N)=X(N)/1000.D+00
      H(1,N)=S(N)/(1.D+00-RHO/1025.D+00)
C
C  ICE SHEET IS DEFINED AS GROUNDED PART OF ICE, ICE SHELF FLOATING
C  FIND GROUNDING LINE
      DO 3 J=1,N
      GRND=-1025.D+00*BED(J)/RHO
      IF(H(1,J).LT.GRND) THEN
        K=J
        GO TO 50
      ENDIF
    3 CONTINUE
   50 K=J
C
C  SET ALL VELOCITIES, DRIVING STRESSES AND BASAL SHEAR STRESSES
C   AT ICE DIVIDE EQUAL TO ZERO
      DO 4 I=1,10
      U(I,1)=0.D+00
      RGH(I,1)=0.D+00
      RGHB(I,1)=0.D+00
      TAUB(I,1)=0.D+00
    4 TAUBB(I,1)=0.D+00
C
C  CALCULATE VELOCITIES FROM CONTINUITY
      U(1,2)=B(2)*DELX/H(1,2)
      DO 5 I=2,M
      HUR=H(1,I)*U(1,I)/R(I)
      BDX=(B(I)-HUR)*2.D+00*DELX
      UH=H(1,I)*U(1,I-1)+U(1,I)*(H(1,I-1)-H(1,I+1))
      U(1,I+1)=(BDX+UH)/H(1,I)
    5 CONTINUE
      DO 11 I=2,N
      WRITE(03,111)U(1,I)
   11 CONTINUE
C
C  CALCULATE SIGMA X
C  CONVERT PASCALS TO BARS FOR OUTPUT STRESSES
      DO 6 J=2,N
      DELU=U(1,J)-U(1,J-1)
      VEL=DELU/SPA
      IF(VEL.GT.0.0) THEN
        SIGX(1,J)=(VEL/(A*DELX))**(1.D+00/3.D+00)
      ENDIF
      IF(VEL.LT.0.0) THEN
        SIGX(1,J)=-1.D+00*(DABS(VEL/(A*DELX)))**(1.D+00/3.D+00)
      ENDIF
      SIGXB(1,J)=SIGX(1,J)/PA
      WRITE(02,111)SIGX(1,J)
  111 FORMAT(F26.19)
    6 CONTINUE
C
C  CALCULATE BASAL DRAG FROM STRESS EQUILIBRIUM FOR ICE SHEET
      DO 7 J=2,M
      SLOPE=(S(J+1)-S(J-1))/(X(J+1)-X(J-1))
      RGH(1,J)=RHO*G*H(1,J)*SLOPE
      RGHB(1,J)=RGH(1,J)/PA*(-1.0D+00)
      DSIGH=SIGX(1,J+1)*(H(1,J)+H(1,J+1))/2.0D+00-SIGX(1,J)*
     $(H(1,J-1)+H(1,J))/2.0D+00
```

```
      TAUB(1,J)=(2.0D+00*DSIGH/DELX)-RGH(1,J)
      TAUBB(1,J)=TAUB(1,J)/PA
    7 CONTINUE
C
C  WRITE RESULTS FOR OUTPUT
      DO 8 J=1,M
      WRITE(1,110)TAUB(1,J)
  110 FORMAT(F30.23)
    8 CONTINUE
      WRITE(6,103)XKM(1),H(1,1),RKM(1),U(1,1),RGHB(1,1),TAUBB(1,1)
  103 FORMAT(1X,F5.0,4X,F7.2,1X,F13.2,6X,F9.3,30X,E12.5,5X,E12.5)
      DO 9 J=2,M
      IF(J.EQ.K) WRITE(6,107)
  107 FORMAT('-------------')
      WRITE(6,104)SIGXB(1,J)
  104 FORMAT(50X,E12.5)
      WRITE(6,103)XKM(J),H(1,J),RKM(J),U(1,J),RGHB(1,J),TAUBB(1,J)
    9 CONTINUE
      WRITE(6,104)SIGXB(1,N)
      WRITE(6,105)XKM(N),H(1,N),RKM(N),U(1,N)
  105 FORMAT(1X,F5.0,4X,F6.1,2X,F9.2,2X,F10.3)
      DO 10 I=1,N
      WRITE(6,207)S(I),BED(I)
  207 FORMAT(F10.2,4X,F10.2)
   10 CONTINUE
      WRITE(6,106)
  106 FORMAT('1')
      STOP
      END
/*
//GO.FT01F001 DD DSN=TS1304.TAUB,
// UNIT=USERDA,
// DISP=(OLD,CATLG,DELETE),
// SPACE=(TRK,(5,2),RLSE),
// DCB=(BLKSIZE=12000,RECFM=FB,LRECL=80)
//GO.FT02F001 DD DSN=TS1304.SIGMAX,
// UNIT=USERDA,
// DISP=(OLD,CATLG,DELETE),
// SPACE=(TRK,(5,2),RLSE),
// DCB=(BLKSIZE=12000,RECFM=FB,LRECL=80)
//GO.FT03F001 DD DSN=TS1304.VEL,
// UNIT=USERDA,
// DISP=(OLD,CATLG,DELETE),
// SPACE=(TRK,(5,2),RLSE),
// DCB=(BLKSIZE=12000,RECFM=FB,LRECL=80)
//GO.SYSIN DD *
/*
//
```

Response Computer Code:

```
//TS1304 JOB 'SDX350,328706683','DONNELLAN',
// REGION=512K,TIME=(3,24)
/*JOBPARM LINES=10000
// EXEC FORTVCG,PARM.FORT='LANGLVL(77)',TIME=(3,24)
//FORT.SYSIN DD *
C
C   RESPONSE AD850405
C
C   CALCULATION OF TIME RESPONSE USING GIVEN THICKNESS
C
      IMPLICIT  DOUBLE PRECISION(A-H,O-Z)
      DIMENSION H(400),BA(400),U(400),XKM(400),RGH(400),X(400),
     +SIGX(400),TAUB(400),RGHB(400),SIGXB(400),TAUBB(400),VEL(400)
      DIMENSION A(400),B(400),C(400),D(400),BED(400),S(400)
      DIMENSION BB(400),Q(400),DHDT(400),RKM(400),R(400)
      DIMENSION HPP(400),HP(400),SIGXP(400),UP(400),SP(400)
      DIMENSION TAUP(400),TAUF(400),DIF(400),TOPA(400),TOPB(400)
      DIMENSION BOTA(400),BOTB(400),SIGPP(400),SF(900),RGHP(400)
C
C   WRITE TITLE OF PROGRAM
      WRITE(6,100)
  100 FORMAT('1','CALCULATION OF NEW THICKNESS USING GIVEN BASAL',
     $' DRAG'/)
C
C LABEL HEADINGS AND WRITE RESULTS FOR OUTPUT
      WRITE(6,101)
  101 FORMAT(' X(KM)        H(M)                  R(KM)    AVERAGE VELOCITY     ',
     $'DEVIATORIC STRESS    DRIVING STRESS      BASAL DRAG          DH/DT')
      WRITE(6,102)
  102 FORMAT(39X,'(M/A)',16X,'(BAR)',14X,'(BAR)',11X,'(BAR)'/)
C
C   VARIABLES ARE DEFINED AS FOLLOWING:  H=THICKNESS,M; BA=ACCUM
C    RATE,M/A; U=VELOCITY,M/A; VEL=VELOCITY,M/S; XKM=HORIZ
C    DIST,KM; X=HORIZ DIST,M; RGH=RHO*G*H,PA; RGHB=RGH,BARS;
C    SIGX=DEVIATORIC STRESS,PA; SIGXB=SIGX,BARS; TAUB=BASAL
C    DRAG,PA; TAUBB=TAUB,BARS
C
C   CONSTANTS DEFINED AS FOLLOWING:  FLOW=FLOW LAW CONST,SE-1*PAE-3;
C    SPA=SEC/YEAR,S; PA=100000BARS; RHO=ICE DENSITY,KG/ME3;
C    G=GRAVITATIONAL ACCEL,M/SE2; DELX=STEP DIST, 25KM
      SPA=3.15D+07
      FLOW =1.8D-25
      PA=100000.0D+00
      RHO=917.0D+00
      G=9.78D+00
      DELX=5000.0D+00
      BE=0.1D+00
      TIME=1.0D+00
C
C   ASSIGN LOOP VARIABLE
      N=1025000.D+00/DELX+1.0D+00
      M=N-1
C
C   FILL XKM AND BA ARRAYS AND CALCULATE THICKNESS AND RADIUS DATA
      STEP=0.0D+00
```

```
      DO 1 I=1,M
      X(I)=DELX*STEP
      XKM(I)=X(I)/1000.0D+00
      BA(I)=0.1D+00
C   CALCULATE SURFACE PROFILE
      SINT=2130.73963753D+00
      SX1=-0.00116124D+00
      SX2=-1.1693194D-07
      SX3=7.4316170D-13
      SX4=-2.0245300D-18
      SX5=2.6264798D-24
      SX6=-1.3227578D-30
      S(I)=SINT+SX1*X(I)+SX2*(X(I)**2.D+00)+SX3*(X(I)**3.D+00)+
     $SX4*(X(I)**4.D+00)+SX5*(X(I)**5.D+00)+SX6*(X(I)**6.D+00)
      IF(X(I).GT.525000.D+00) S(I)=-.1058D-03*X(I)+140.1D+00
C   CALCULATE BED PROFILE
      IF(X(I).LT.50000D+00) THEN
         BINT=2.2343674D-11
         BX1=-0.02120093D+00
         BX2=2.2354167D-06
         BX3=-9.8762255D-11
         BX4=1.0906863D-15
         BED(I)=BINT+BX1*X(I)+BX2*(X(I)**2.D+00)+BX3*(X(I)**3.D+00)+
     $   BX4*(X(I)**4.D+00)
      ENDIF
      IF(X(I).GE.50000.D+00) BED(I)=-1000.D+00
      IF(X(I).GT.125000.D+00) BED(I)=.02D+00*X(I)-3500.
      IF(X(I).GT.150000.D+00) THEN
         BINT=20276.27011274D+00
         BX1=-0.40475457D+00
         BX2=3.1371758D-06
         BX3=-1.2514167D-11
         BX4=2.7092624D-17
         BX5=-3.0199322D-23
         BX6=1.3563988D-29
         BED(I)=BINT+BX1*X(I)+BX2*(X(I)**2.D+00)+BX3*(X(I)**3.D+00)+
     $   BX4*(X(I)**4.D+00)+BX5*(X(I)**5.D+00)+BX6*(X(I)**6.D+00)
      ENDIF
      IF(X(I).GT.397000.D+00) BED(I)=-700.D+00
      SF(I)=BED(I)*(1.0D+00-1025.D+00/RHO)
      H(I)=S(I)-BED(I)
      IF(S(I).LE.SF(I)) H(I)=S(I)/(1.D+00-RHO/1025.D+00)
C   CALCULATE RADIUS OF CURVATURE
         RINT=-1024550.08443342D+00
         RX1=8.00708578D+00
         RX2=-0.00014123D+00
         RX3=2.8648497D-09
         RX4=-2.3618092D-14
         RX5=8.5037776D-20
         RX6=-1.1514543D-25
         R(I)=RINT+RX1*X(I)+RX2*(X(I)**2.D+00)+RX3*(X(I)**3.D+00)+
     $   RX4*(X(I)**4.D+00)+RX5*(X(I)**5.D+00)+RX6*(X(I)**6.D+00)
      IF(X(I).GE.275000.D+00) R(I)=-1000000.D+00
      IF(X(I).GE.300000.D+00) R(I)=-2000000.D+00
      IF(X(I).GE.325000.D+00) R(I)=-20000000.D+00
      IF(X(I).GE.350000.D+00) R(I)=-200000000.D+00
      R(I)=R(I)/2.D+00
      RKM(I)=R(I)/1000.D+00
      STEP=STEP+1.0D+00
 1    CONTINUE
```

```
      BED(N)=2.D+00*BED(M)-BED(M-1)
      S(N)=2.D+00*S(M)-S(M-1)
      H(N)=S(N)/(1.D+00-RHO/1025.D+00)
      R(N)=-200000000.D+00/2.D+00
      RKM(N)=R(N)/1000.D+00
      X(N)=DELX*STEP
      XKM(N)=X(N)/1000.D+00
C
C ICE SHEET IS DEFINED AS GROUNDED PART OF ICE, ICE SHELF FLOATING
C FIND GROUNDING LINE
      DO 3 J=1,N
      GRND=-1025.D+00*BED(J)/RHO
      IF(H(J).LT.GRND) THEN
        KGRND=J
        GO TO 50
      ENDIF
  3   CONTINUE
 50   KGRNDP=KGRND
C
C READ BASAL DRAG, SIGMAX, VELOCITY
      DO 2 I=1,M
      READ(2,201)TAUB(I)
      READ(3,111)SIGX(I+1)
      READ(4,111)U(I+1)
      SIGXB(I+1)=SIGX(I+1)/PA
 201  FORMAT(F30.23)
 111  FORMAT(F26.19)
      TAUBB(I)=TAUB(I)/PA
      SIGXP(I+1)=SIGX(I+1)
      SIGXB(I+1)=SIGX(I+1)/PA
  2   CONTINUE
      DO 9 I=1,N
  9   HPP(I)=H(I)
      FORCE=SIGX(2)*(H(1)+H(2))/2.D+00
      FORCEP=FORCE
C
C ESTABLISH FINAL VALUES OF RESISTIVE DRAG
      DO 22 I=2,M
 22   TAUF(I)=TAUB(I)
      DO 31 I=1,24
      READ(8,201)TAUF(I+88)
 31   CONTINUE
C
      DO 16 I=2,M
      SLOPE=(S(I+1)-S(I-1))/(X(I+1)-X(I-1))
      RGH(I)=RHO*G*H(I)*SLOPE
 16   RGHB(I)=RGH(I)/PA*(-1.D+00)
      RGH(N)=RHO*G*H(N)*(S(N)-S(M))/DELX
      RGHB(N)=RGH(N)/PA*(-1.D+00)
C
C PRINT INITIAL ICE SHEET PROFILE
      WRITE(6,118)
 118  FORMAT(1X,'INITIAL PROFILE',/)
      WRITE(6,120)XKM(1),H(1),RKM(1),U(1),RGHB(1),TAUBB(1)
 120  FORMAT(1X,F5.0,4X,F9.4,3X,F13.2,2X,F10.3,30X,F12.7,5X,F12.7)
      DO 19 I=5,M,4
      WRITE(6,104)SIGXB(I-1)
      WRITE(6,120)XKM(I),H(I),RKM(I),U(I),RGHB(I),TAUBB(I)
      IF(I.LT.KGRND) THEN
        IF(KGRND.LE.I+4) WRITE(6,206)
```

```
  206 FORMAT('-------------')
      ENDIF
  19  CONTINUE
      WRITE(6,104)SIGXB(N-2)
      WRITE(6,119)XKM(N),H(N),RKM(N),U(N),RGHB(N)
  119 FORMAT(1X,F5.0,4X,F9.4,3X,F13.2,2X,F10.3,30X,F12.7)
      WRITE(6,116)
C
      DO 75 K=1,10
C
C   CALCULATE NEW RESISTIVE DRAG
      DO 24 I=1,N-1
      TAUP(I)=TAUF(I)
  24  CONTINUE
C
      KGRND=KGRNDP
C
  14  DO 12 I=1,N
      HP(I)=HPP(I)
      IF(I.LT.KGRND) SP(I)=HP(I)+BED(I)
  12  IF(I.GE.KGRND) SP(I)=HP(I)*(1-RHO/1025.D+00)
C
C   CALCULATE DRIVING STRESSES
      DO 17 I=2,M
      SLOPE=(S(I+1)-S(I-1))/(X(I+1)-X(I-1))
      SLOPEP=(SP(I+1)-SP(I-1))/(X(I+1)-X(I-1))
      RGH(I)=RHO*G*(H(I)*SLOPE+HP(I)*SLOPEP)/2.D+00
  17  RGHB(I)=RGH(I)/PA*(-1.0D+00)
      SLOPE=(S(N)-S(M))/DELX
      SLOPEP=(SP(N)-SP(M))/DELX
      RGH(N)=RHO*G*(H(N)*SLOPE+HP(N)*SLOPEP)/2.D+00
      RGHB(N)=RGH(N)/PA*(-1.0D+00)
C
C   SET ALL VELOCITIES, DRIVING STRESSES AND BASAL SHEAR STRESSES
C    AT ICE DIVIDE EQUAL TO ZERO
      U(1)=0.0D+00
      RGH(1)=0.0D+00
      RGHB(1)=0.0D+00
      TAUB(1)=0.0D+00
      TAUP(1)=0.0D+00
      TAUBB(1)=0.0D+00
      UP(1)=0.0D+00
C
C   USE GIVEN BASAL DRAG TO CALCULATE NEW DEVIATORIC STRESSES ON
C    ICE SHEET
      SIGX(2)=FORCE/((H(1)+H(2))/2.D+00)
      SIGPP(2)=FORCEP/((HPP(1)+HPP(2))/2.D+00)
      SIGXP(2)=SIGPP(2)
      DO 8 I=2,M
      GRAD=((TAUB(I)+TAUP(I))/2.D+00+RGH(I))*2.D+00*DELX
      HTS=(H(I)+H(I-1))*SIGX(I)+(HP(I)+HP(I-1))*SIGPP(I)
      STH=SIGX(I+1)*(H(I+1)+H(I))
      HPH=HP(I+1)+HP(I)
      SIGPP(I+1)=(HTS+GRAD-STH)/HPH
      SIGXB(I+1)=SIGPP(I+1)/PA
   8  CONTINUE
C
C   CALCULATE VELOCITIES FROM THE ICE DIVIDE TO GROUNDING LINE
C    USE FLOW LAW
      SUM=0.0D+00
```

```
      DO 10 J=2,N
C   INTEGRATE (SIGMAX)**(3)*DELTA X
      IF (SIGPP(J).LT.0) THEN
        SIGPP(J)=SIGPP(J)*(-1.0D+00)
        CUBE=SIGPP(J)**3.D+00
        CUBE=CUBE*(-1.0D+00)
      ENDIF
      IF (SIGPP(J).GE.0) THEN
        CUBE=SIGPP(J)**3.D+00
      ENDIF
      SUM=SUM+CUBE*DELX
C   CALCULATE VELOCITIES
      VEL(J)=FLOW*SUM
   10 UP(J)=VEL(J)*SPA
C
C   USE FORWARD DIFFERENCING TO COMPUTE NEW THICKNESSES
C   KEEP ANNUAL ACCUMULATION CONSTANT BE=0.1
C   TIME STEP HAS BEEN DEFINED AS 1 YEAR
      DHDT(1)=BE-((H(2)*U(2)/DELX)+(HP(2)*UP(2)/DELX))/2.D+00
      HPP(1)=DHDT(1)*TIME+H(1)
      DO 11 I=2,M
      HDU=(HP(I)*(UP(I+1)-UP(I-1))+H(I)*(U(I+1)-U(I-1)))/DELX
      UDH=(UP(I)*(HP(I+1)-HP(I-1))+U(I)*(H(I+1)-H(I-1)))/DELX
      HUR=(H(I)*U(I)+HP(I)*UP(I))/(2.D+00*R(I))
      DHDT(I)=-(HDU+UDH)/4.D+00-HUR+BE
      HPP(I)=DHDT(I)*TIME+H(I)
   11 CONTINUE
      IF(M.LT.KGRND) SP(M)=BED(M)+HPP(M)
      IF(M.GE.KGRND) SP(M)=HPP(M)*(1-RHO/1025.D+00)
      IF(M-1.LT.KGRND) SP(M-1)=BED(M-1)+HPP(M-1)
      IF(M-1.GE.KGRND) SP(M-1)=HPP(M-1)*(1-RHO/1025.D+00)
      SP(N)=(2.D+00*SP(M))-SP(M-1)
      IF(N.LT.KGRND) HPP(N)=SP(N)-BED(N)
      IF(N.GE.KGRND) HPP(N)=SP(N)/(1-RHO/1025.D+00)
      DHDT(N)=(HPP(N)-H(N))/TIME
C
      DO 25 I=1,N
      HP(I)=HPP(I)
      IF(I.LT.KGRND) SP(I)=HP(I)+BED(I)
      IF(I.GE.KGRND) SP(I)=HP(I)*(1-RHO/1025.D+00)
   25 CONTINUE
C
C   CALCULATE DRIVING STRESSES FOR OUTPUT
   30 DO 23 I=2,M
      SLOPEP=(SP(I+1)-SP(I-1))/(X(I+1)-X(I-1))
      RGH(I)=RHO*G*(HPP(I)*SLOPEP)
   23 RGHB(I)=RGH(I)/PA*(-1.0D+00)
      SLOPEP=(SP(N)-SP(M))/DELX
      RGH(N)=RHO*G*(HPP(N)*SLOPEP)
      RGHB(N)=RGH(N)/PA*(-1.0D+00)
C
      DO 13 I=1,N
      DEL=DABS((HPP(I)-HP(I))/HP(I))
      IF(DEL.GT.0.01D+00) GO TO 14
   13 CONTINUE
C
C   ICE SHEET IS DEFINED AS GROUNDED PART OF ICE, ICE SHELF FLOATING
C   FIND GROUNDING LINE
      DO 26 J=1,N
      GRND=-1025.D+00*BED(J)/RHO
```

```
      IF(H(J).LT.GRND) THEN
        KGRNDP=J
        GO TO 51
      ENDIF
   26 CONTINUE
   51 KGRNDP=J
C
      DO 70 J=1,10
      IF(K.EQ.J) THEN
        L=J*TIME
        WRITE(6,115)L
  115   FORMAT(1X,I5,' YEARS',/)
      H(1)=HPP(1)
      DO 15 I=2,N
      U(I)=UP(I)
      TAUB(I-1)=TAUP(I-1)
      TAUBB(I-1)=TAUB(I-1)/PA
      SIGX(I)=SIGPP(I)
      SIGXB(I)=SIGX(I)/PA
   15 H(I)=HPP(I)
      WRITE(6,103)XKM(1),H(1),RKM(1),U(1),RGHB(1),TAUBB(1),DHDT(1)
  103 FORMAT(1X,F5.0,4X,F9.4,3X,F13.2,2X,F10.3,30X,F12.7,5X,F12.7,F15.7)
      DO 18 I=60,M
      WRITE(6,104)SIGXB(I)
  104 FORMAT(50X,F12.7)
      WRITE(6,103)XKM(I),H(I),RKM(I),U(I),RGHB(I),TAUBB(I),DHDT(I)
      IF(I.LT.KGRNDP) THEN
        IF(KGRNDP.LE.I+1) WRITE(6,203)
  203 FORMAT('-------------')
      ENDIF
   18 CONTINUE
      WRITE(6,104)SIGXB(N)
      WRITE(6,117)XKM(N),H(N),RKM(N),U(N),RGHB(N),DHDT(N)
  117 FORMAT(1X,F5.0,4X,F9.4,3X,F13.2,2X,F10.3,30X,F12.7,17X,F15.7)
        WRITE(6,116)
  116   FORMAT(/)
      ENDIF
   70 CONTINUE
      H(1)=HPP(1)
      IF(1.LT.KGRND) S(1)=H(1)+BED(1)
      IF(1.GE.KGRND) S(1)=H(1)*(1-RHO/1025.D+00)
      DO 21 I=2,N
      U(I)=UP(I)
      TAUB(I-1)=TAUP(I-1)
      FORCE=FORCEP
      H(I)=HPP(I)
      IF(I.LT.KGRND) S(I)=H(I)+BED(I)
      IF(I.GE.KGRND) S(I)=H(I)*(1-RHO/1025.D+00)
   21 SIGX(I)=SIGPP(I)
   75 CONTINUE
      WRITE(6,105)
  105 FORMAT('1')
      STOP
      END
/*
//GO.FT02F001 DD DSN=TS1304.TAUB,DISP=SHR
//GO.FT03F001 DD DSN=TS1304.SIGMAX,DISP=SHR
//GO.FT04F001 DD DSN=TS1304.VEL,DISP=SHR
//GO.FT08F001 DD DSN=TS1304.NEWTAU,DISP=SHR
//GO.SYSIN DD *
```

References:

Budd, W. F. and McInnes B. J., Periodic Surging of the Antarctic Ice Sheet--An Assessment by Modelling, Hydrological Sciences Bulletin 24, no. 1, 95-104, 1979.

Hughes, T., Is the West Antarctic Ice Sheet Disintegrating? J. Geophysical Research 78, no. 33, 7784-7909, 1975.

Jankowski, E. J. and Drewry, D. J. The Structure of West Antarctica from Geophysical Studies, Nature, 291, 17-21, 1981.

Lingle, C. S., A Numerical Model of Interactions Between a Polar Ice Streams and the Ocean: Application to Ice Stream E, West Antarctica, J. of Geophysical Research, 81, 3523-3549, 1984.

McInnes, B. J., Budd, W. F., Radok U., Cross-sectional Models for West Antarctica and Greenland, presented at Symposium on Ice and Climate Modelling, Evanston, 27 June to 1 July, 1983.

Mercer, J. H., West Antarctic Ice Sheet and CO_2 Greenhouse Effect: A Threat of Disaster, Nature, 271, no. 5643, 321-325, 1978.

Paterson, W. S. B. in The Physics of Glaciers, New York, 31, 1983.

Thomas, R. H. and Bentley, C. R., A Model for Holocene Retreat of the West Antarctic Ice Sheet, Quar. Res. 10, 150-170, 1978.

van der Veen, C. J. Response of a Marine Ice Sheet to Changes at the Grounding Line, Quat. Res. 24, 257-267, 1985.

Van Heeswijk, M. Meteorite Impact by Ice Flow, B.Sc. Thesis, Dept. of Geol. and Min., Ohio State University, 35-37, 1983.

Whillans, I. M. Radio Echo Layers and the Recent Stability of the West Antarctic Ice Sheet, Nature, 264, no. 5582, 152-155, 1976.

Whillans, I. M. Personal communication, 1986.

Figures

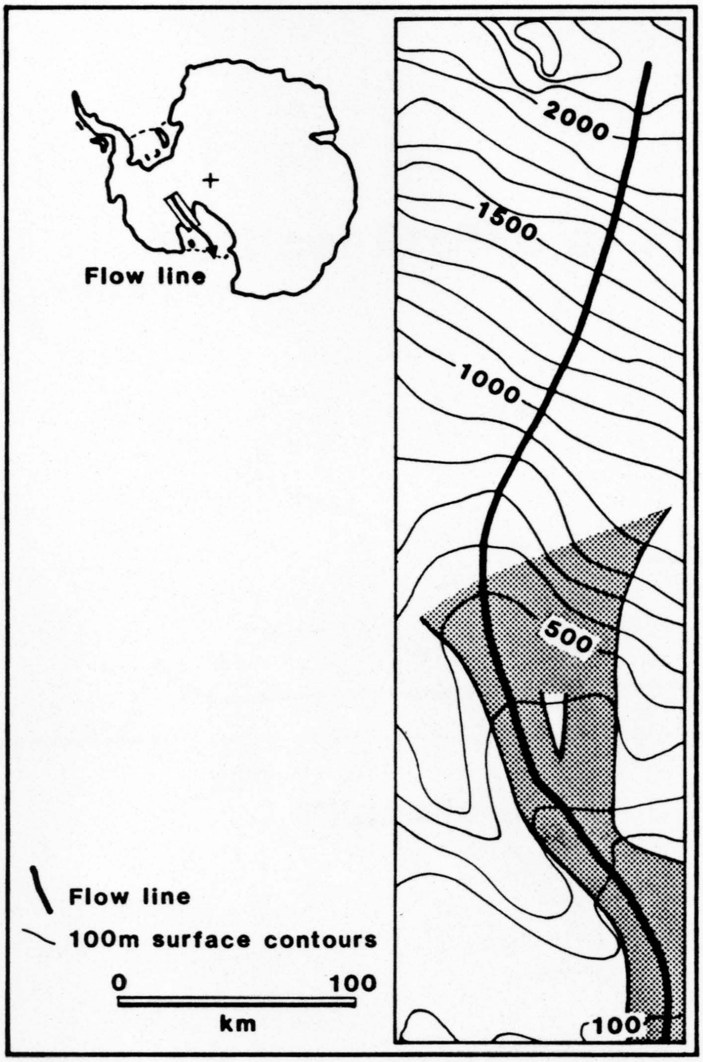


Fig. 1. Map view of flow line used for calculations. Complete flow line extends to the end of the ice shelf (after Jankowski and Drewry, 1981).

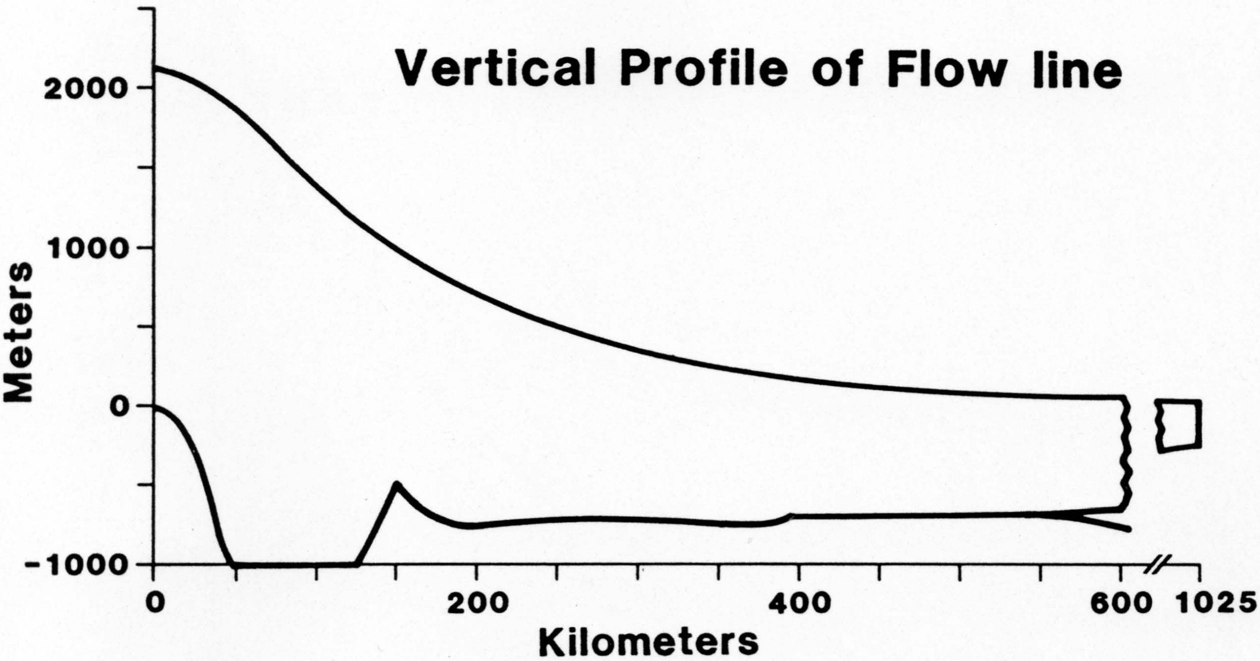


Fig. 2. Bed and surface elevations for the present ice sheet.

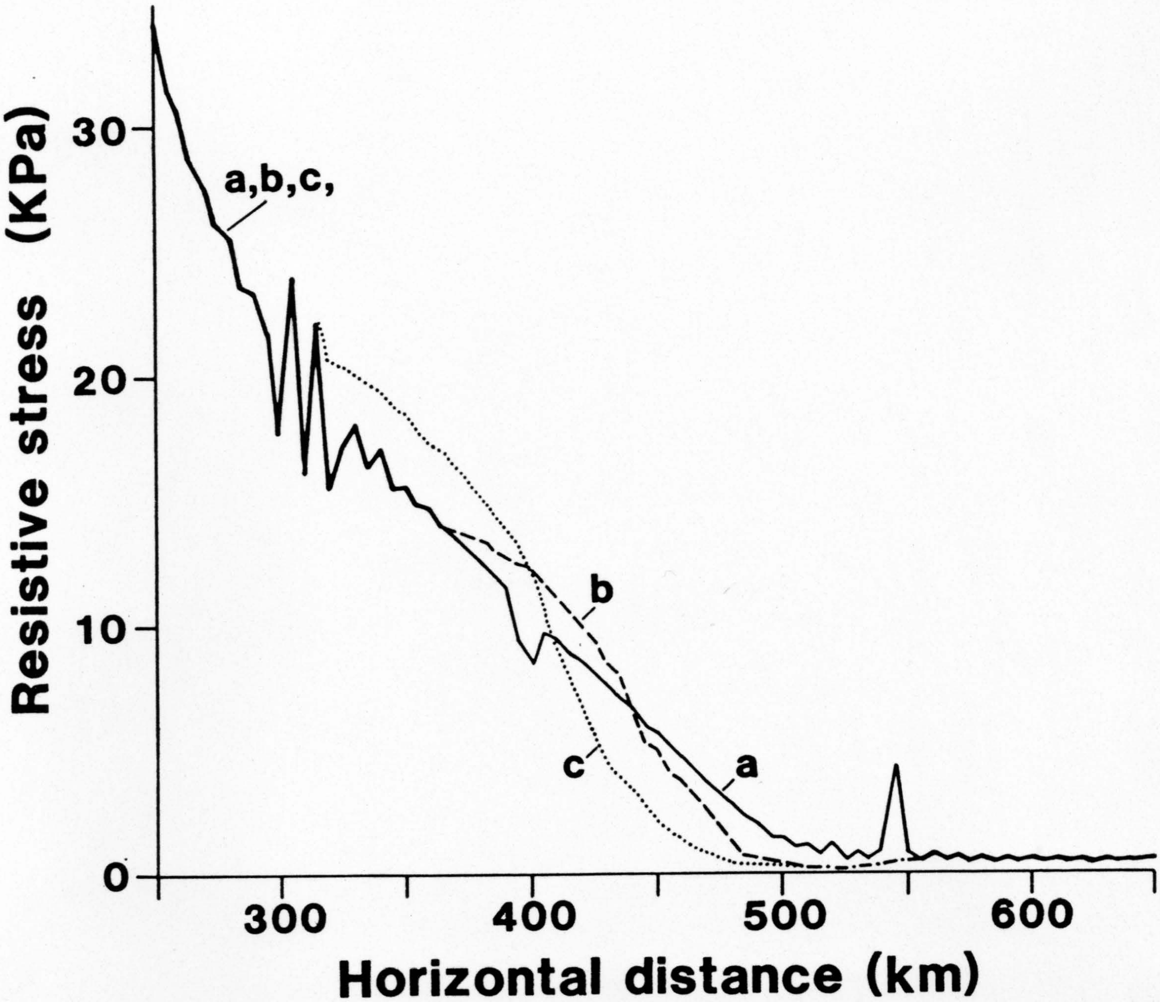


Fig. 3. Distributions of resistive drag just up-glacier from the grounding line: a) initial stress distribution calculated for the present ice sheet, b) redistribution of resistive drag used to simulate a rise in sea level, c) further redistribution of resistive drag to test if the response is stable. Spikes occur at transitions between inland ice and ice stream and between ice stream and ice shelf. They are much reduced with smaller grid spacing and have no bearing on final results.

Tables

Table 1. Startup Calculation-Flow Chart

1. Ice sheet is in equilibrium for initialization.

$$\frac{\partial H}{\partial t} = 0$$

2. Set velocity at divide to zero.

$$\bar{U}_{divide} = 0$$

3. Calculate U at first grid point.

$$\bar{U}_1 = \frac{\dot{b}_1}{H_1}\Delta x$$

4. Calculate velocities from divide to grounding line.

$$\bar{U}_{i+1} = (\dot{b} - \frac{H_i \bar{U}_i}{R_i})2\Delta x - \bar{U}_i(H_{i+1} - H_{i-1}) + \bar{U}_{i-1}$$

5. Calculate stretching stresses.

$$\sigma_x' = (\frac{\bar{U}_i - \bar{U}_{i-1}}{A\Delta x})^{\frac{1}{3}}$$

6. Calculate resistive drag.

$$\tau_r = \rho g H_i((h_{i+1} - h_{i-1})2\Delta x)$$

$$+ ((H_i + H_{i+1})\sigma'_{xi+\frac{1}{2}} - H_{i-1} + H_i)\sigma'_{xi+\frac{1}{2}}) / \Delta x$$

Symbol definitions:

$\bar{U}$ - verticallly averaged velocity
H - thickness
$\dot{b}$ - annual accumulation rate
R - radius of curvature of surface contours
A - softness parameter
σ_x' - stretching stress
ρ - ice density
g - gravitational acceleration
h - surface elevation
τ_r - resistive drag

Table 2. Response Calculation-Flow Chart

1. Specify from startup procedure.

   τ - resistive drag
   H - thickness
   h - surface elevation
   $\bar{U}$ - mean velocity
   σ_x' - stretching stress
   τ_r' - new distribution of resistive drag

   Prime (') indicates new estimate for time averaging.
   Double prime ('') indicates best estimate for time averaging.

2. Establish first estimates of quantities.

$$\bar{U}' = U$$
$$H' = H$$
$$h' = h$$
$$\sigma_x'' = \sigma_x'$$

3. Hold stretching stress at ice divide or calving front constant as a boundary condition.

4. Solve stress equilibrium for σ_x'' by averaging over both distance and time.

$$\sigma''_{x\,i-\frac{1}{2}} = \Big((H_{i+1} + H_i)\sigma'_{x\,i+\frac{1}{2}} - (H_{i-1}+H_i)\sigma'_{x\,i-\frac{1}{2}} + (H'_{i+1}+H_i')\sigma''_{x\,i+\frac{1}{2}}$$
$$-(\tau_i+\tau'_i)\Delta x + \frac{1}{4}\rho g(H'_i+H_i)(h_{i+1}-h_{i-1}+h'_{i+1}-h'_{i-1})\Big)/(H'_{i-1}+H'_i)$$

5. Solve flow law for $\bar{U}''$.

$$\bar{U}'_{i+1} = (A\sigma''^{\,3}_{x\,i+\frac{1}{2}})\Delta x+\bar{U}_i$$

6. Solve continuity for $\partial H/\partial t$.

$$\frac{\partial H}{\partial t} = \frac{(H_i+H'_i)(\bar{U}_{i+1}-\bar{U}_{i-1}+\bar{U}'_{i+1}-\bar{U}'_{i-1})}{8\Delta x} + \frac{(\bar{U}_i+\bar{U}'_i)(H_{i+1}-H_{i-1}+H'_{i+1}-H'_{i-1})}{8\Delta x}$$
$$-\frac{(H_i\bar{U}_i+H'_i\bar{U}'_i)}{2R_i} + B_i$$

7. Calculate improved thickness, H''.

$$H'' = H+\frac{\partial H}{\partial t}\Delta t$$

8. Compare H' to H''. If change is greater than 1% go to step 3 with new H', σ_x'', $\bar{U}'$, h'.

9. Advance one time step, go to step 2.

Table 3. Results for 36% Change in Resistive Force*

	CHANGES IN ICE SHEET			
Model	Inland	Near Grounding Line	Ice Shelf	Time Response
A	-No change	-Inc. τ_r →inc. surface slope	-Dec. surface slope	-80a to reach 1/e of final changes
Secondary effects on ice shelf		-Dec. τ_r →dec. surface slope	-Thinner near grounding line	-200a to reach 90% of final changes
		-Thinner	-Thicker by .5 at calving front	-60a for grounding line to stabilize
		-Max. thinning 1.8% (14.6 m)		
		-45 m grounding line retreat		
B	-No impt. changes	-Inc. τ_r →inc. surface surface slope	-Dec. surface slope	-80a to reach 1/e of final changes
Secondary effects on inland ice	-Thinner by .002-.01%	-Dec. τ_r →dec. surface slope	-Thinner near grounding line	-200a to reach 90% of final changes
		-Thinner	-Thicker by .02% at at calving	-60a for grounding line to stabilize
		-Max. thinning 1.8% (14.6 m)		
		-45 m grounding line retreat		

*Reduction covers 115 km zone